Mechanics Corner

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Planetary Gear Challenge

1 Introduction

Back in the mid-1990s, I worked as the "in-house consultant" for a small aerospace manufacturing firm in the Chicago suburbs (500 employees). The general rule was simply that any engineer in the company could bring me any problem, and I would try to give them a solution. It was one of the happiest jobs I've ever had because I was constantly being given new problems to work on. Some of the problems were simple, the sort that could be solved in 20 minutes with pencil and paper. Others took many days and lots of computer work, but I had a free hand and I really enjoyed it.

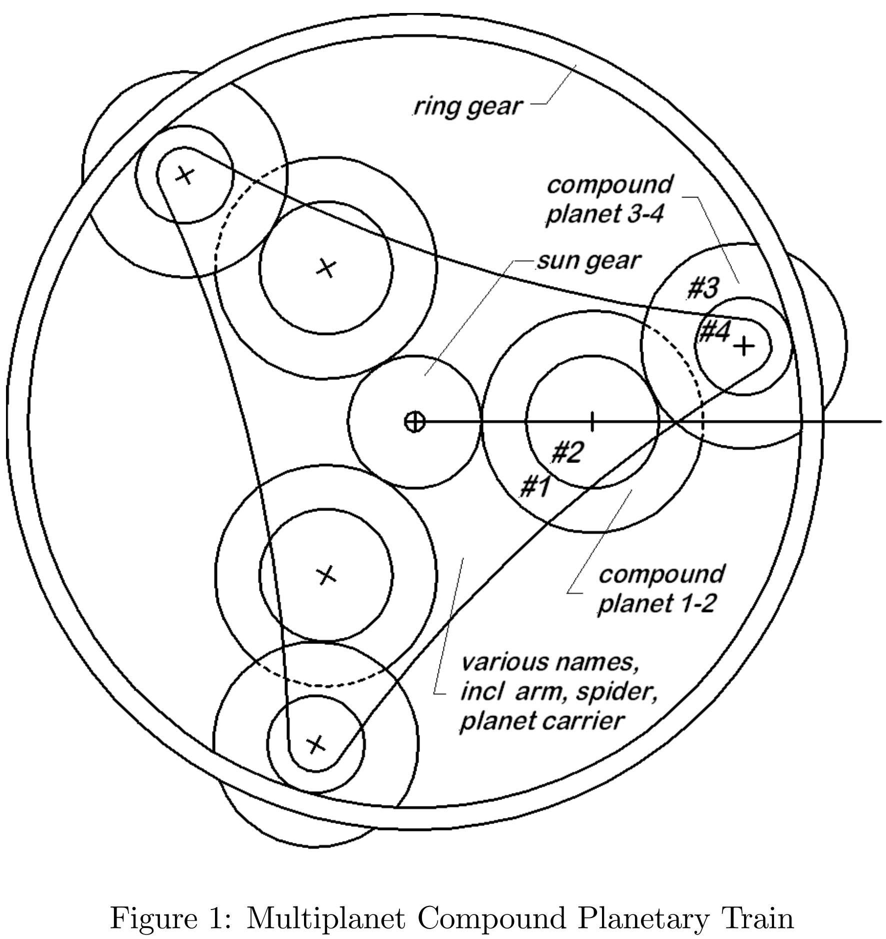

Figure 1: Multiplanet Compound Planetary Train

The company build components for use in commercial and military aircraft and in spacecraft. They specialized in very small electric motors and gear trains to actuate control surfaces and similar applications. A very small electric motor, not as large as your fist but running at very high speed (say 10,000 rpm), when geared down can produce considerable torque or force. This usually requires very large speed reductions, and for that purpose, the company built a lot of gear trains.

One of the gear train types we made is pictured in Figure 1. The small electric motor drives the sun gear, and in most cases the output is taken from the concentric planet carrier shaft. This brings us to the topic for this challenge, planetary gear trains with multiple compound planets in the load path.

2 Planetary Trains

Planetary trains typically involve a sun gear (in the center), an array of planet gears supported in a frame called the planet carrier, and a ring gear that surrounds the entire assembly. There are many variations on this idea, but these are the common elements. If the planet gears are compound gears, this means that two gears are mounted on a common shaft and rotate together. In the system shown in Figure 1, there are two compound planets in each load path. As far as the kinematics are concerned, only one branch needs to be considered, say the sun gear, the compound planet 1-2, the compound planet 3-4, and the ring gear. The other branches are kinematically identical to the first one.

The circles shown in Figure 1 represent the gear pitch circles. From a kinematic perspective, the system functions like a collection of smooth, solid disks that roll on each other. Thus, for kinematic analysis, it is the pitch radii that are important. Even so, gears are almost always specified by giving the number of teeth on the gear and the module (the module describes the size of the tooth). For any numerical work in the questions below, assume that the module is $m = 2$ mm/T, for which the pitch radius in millimeters is equal to the number of teeth. The tooth numbers to be considered are these:

$N_s = 16$	sun gear
$N_1 = 29$	inner planet gear
$N_2 = 17$	inner planet gear
$N_3 = 35$	outer planet gear
$N_4 = 17$	outer planet gear
$N_r = 107$	ring gear

In the questions that follow, you may work in terms of the gear tooth numbers $(N_s, N_1, N_2, N_3, N_4, N_r)$ or the pitch radii $(R_s, R_1, R_2, R_3, R_4, R_r)$ since they are completely interchangeable for this module value.

Planetary trains are *two degree of freedom mechanisms*. In the long-ago era of mechanical analog computing (useful for bomb sights and naval artillery trajectory calculations), this allowed planetary trains to be used for addition and subtraction in the calculations. Yet today, the differential gear at the center of the automobile rear axel is a planetary gear set.

In the planetary context, it is sufficient to say that this means that there are three coupled variables, and two of them must be specified in order to determine the third. The three variables are

ω_s = sun gear angular velocity

ω_{pc} = planet carrier angular velocity

ω_r = ring gear angular velocity

where all angular velocities are measured with respect to a stationary reference.

It is common to work in terms of angular velocities to avoid the question of initial values. Otherwise, the associated angles θ_s, θ_{pc},and θ_r could be used just as well. Note that, in principle, ω_{12} and/or ω_{34} could replace one of the three variables specified. As a practical matter, this is rarely done because these rotations are physically in accessible because of their move shaft centerlines.

3 Challenge Questions

1. Working in symbols only (no numeric values), develop the mathematical relation between ω_s, ω_{pc}, and ω_r, for the train shown in Figure 1.

2. The term train ratio is usually understood to mean $\omega_{output}/\omega_{input}$. In the present case, because this is a speed reduction gear, the train ratio defined in that fashion is a number between -1 and $+1$. This compresses the result into a narrow range. For a reduction gear, it is often more useful to look at $\omega_{input}/\omega_{output}$ a number that is always of magnitude greater than 1.

   (a) If the ring gear is considered fixed and the input is the sun gear rotation, express the value ω_{pc}/ω_s symbolically.

   (b) Using the numeric values provided, what is the value of the ratio in part (a)?

   (c) Does the planet carrier rotate in the same direction as the sun gear or the opposite?

3. Consider a case in which the ring gear is taken as the output. The sun gear is driven at 3500 rpm clockwise while the planet carrier is driven at 1575 rpm counter clockwise. At what speed, and in what direction does the ring gear rotate?

4. In order to create an accurate dynamical model for the gear train using an energy based approach, it is necessary to know the angular velocities of all elements in symbolic form. How are ω_{12} and ω_{34} expressed in terms of ω_s and ω_{pc}? (We will assume that the mass moments of inertia for these elements are know in order to complete the dynamical model.)